International Journal of Theoretical Physics, Vol. 31, No. 3, 1992

Three-Particle Einstein–Podolsky–Rosen Correlations

K. Wódkiewicz[1,2]

Received June 14, 1991

It has been argued by Mermin that a *gedanken* decay of a three-particle system gives a more powerful demonstration of quantum nonlocality than Bell's analysis. It is shown that this claim is premature. An *ad hoc* model based on local realism is constructed in order to reproduce the quantum mechanical prediction of the three-particle *gedanken* decay.

In recent publications (Greenberger *et al.*, 1989; Mermin, 1990*a–c*) it has been suggested that a more powerful refutation of local hidden variables (LHV) or local reality (LR) can be accomplished if instead of a series of two-particle Einstein–Podolsky–Rosen (EPR) correlation experiments one performs only a single measurement involving four or three spin-1/2 particles in a *gedanken* decay.

In a nice and simple description of this three-particle EPR correlation by Mermin (1990*a,b*), the failure of LR has been attributed to a crucial minus sign emerging from an odd number of anticommutations of Pauli matrices.

It is assumed in these references that the correlated state of the three spin-1/2 particles, named a, b, and c, has the following form:

$$|\psi\rangle=\frac{1}{\sqrt{2}}\left(|+_a, +_b, +_c\rangle-|-_a, -_b, -_c\rangle\right) \tag{1}$$

where $|+\rangle$ and $|-\rangle$ specify spin up or down along an arbitrarily chosen axis z.

[1]Center for Advanced Studies and Department of Physics and Astronomy, University of New Mexico, Albuquerque, New Mexico 87131.
[2]Permanent address: Institute of Theoretical Physics, Warsaw University, Warsaw 00-681, Poland.

0020-7748/92/0300-0515$06.50/0

This correlated state has the property that consecutive measurements of the x component and two y components described by the three quantum mechanical observables

$$\hat{O}_1 = \sigma_{ax}\sigma_{by}\sigma_{cy} \tag{2a}$$

$$\hat{O}_2 = \sigma_{ay}\sigma_{bx}\sigma_{cy} \tag{2b}$$

$$\hat{O}_3 = \sigma_{ay}\sigma_{by}\sigma_{cx} \tag{2c}$$

lead to equal expectation values which are opposite in sign to a single measurement of the three x components represented by the quantum observable

$$\hat{O} = \sigma_{ax}\sigma_{bx}\sigma_{cx} \tag{3}$$

We have in this case

$$\langle\hat{O}_1\rangle = \langle\hat{O}_2\rangle = \langle\hat{O}_3\rangle = -\langle\hat{O}\rangle \tag{4}$$

This result can be simply derived by noting that, due to the odd number of anticommutators of the Pauli matrices, we have

$$\hat{O}_1\hat{O}_2\hat{O}_3 = -\hat{O} \tag{5}$$

and that $|\psi\rangle$ is an eigenstate of the three observables $\hat{O}_1$, $\hat{O}_2$, and $\hat{O}_3$.

It has been argued by Mermin (1990*a*,*b*) that the magic of quantum mechanics involving an odd number of anticommutations in equation (5) produces the relation (4) that refutes LR in an always vs. never way and is devastating for the LHV theories.

In this paper I argue that the three-particle *gedanken* decay does not, unfortunately, refute all possible LR assumptions or all possible LHV theories as Bell's inequalities do. The power of the Bell inequalities follows from the fact that by a properly selected series of measurements involving spin correlations, one can rule out once and for ever all possible LR or LHV theories. According to Mermin (1990*a*,*b*), the three-particle decay provides an example in which "the elements of reality require a class of outcomes to occur all the time, while quantum mechanics never allows them to occur." I shall argue that this property is absent in the three-particle decay and that different models based on LR can reproduce the result given by equation (4).

It is, of course, true that the particular model of LR investigated by Mermin (1990*a*,*b*) is incompatible with equation (4), but the entire description is vulnerable, and other *ad hoc* constructed models of LR can reproduce the quantum result.

In the following I give an explicit construction of such an *ad hoc* model of LHV that reproduces equation (4). For the simplicity of the argument I restrict the description of the spin to only x and y components. Let us assume that these components are described by objective local realities $m_{ix}(\lambda_i)$ and $m_{iy}(\lambda_i)$, each dependent on some local unspecified hidden variables λ_i ($i = a, b, c$). In this description the magic of quantum mechanics is removed entirely, because there are no commutators, observables, eigenvalues, or anticommutators. We are just left with statistical averages of the six objective realities. In such a theory correlations are described by the following expressions:

$$O_{1\text{LHV}} = \int d\lambda_a \int d\lambda_b \int d\lambda_c \, P(\lambda_a; \lambda_b; \lambda_c) m_{ax}(\lambda_a) m_{by}(\lambda_b) m_{cy}(\lambda_c) \tag{6a}$$

$$O_{2\text{LHV}} = \int d\lambda_a \int d\lambda_b \int d\lambda_c \, P(\lambda_a; \lambda_b; \lambda_c) m_{ay}(\lambda_a) m_{bx}(\lambda_b) m_{cy}(\lambda_c) \tag{6b}$$

$$O_{3\text{LHV}} = \int d\lambda_a \int d\lambda_b \int d\lambda_c \, P(\lambda_a; \lambda_b; \lambda_c) m_{ay}(\lambda_a) m_{by}(\lambda_b) m_{cx}(\lambda_c) \tag{6c}$$

$$O_{\text{LHV}} = \int d\lambda_a \int d\lambda_b \int d\lambda_c \, P(\lambda_a; \lambda_b; \lambda_c) m_{ax}(\lambda_a) m_{bx}(\lambda_b) m_{cx}(\lambda_c) \tag{6d}$$

where the distribution $P(\lambda_a; \lambda_b; \lambda_c)$ depends locally on the three sets of hidden variables λ_a, λ_b, and λ_c. This distribution is positive and normalized:

$$\int d\lambda_a \int d\lambda_b \int d\lambda_c \, P(\lambda_a; \lambda_b; \lambda_c) = 1 \tag{7}$$

A particular model (Mermin, 1990*a*,*b*) of LR failed to reproduce equation (4) because of a wrong sign. I give now a different model of LHV that reproduces equation (4) with the correct minus sign without ever involving anticommutators, i.e., objects completely foreign to a local and objective description of spin correlations.

This model is based on the classical Malus law for the transmission of light through a linear polarizer. If an electric field with polarization **p** impinges on a polarizer in a direction **a**, the transmitted electric field is attenuated by a cosine function dependent only on the relative angle between the directions of **a** and **p**. Let us apply this classical Malus law to the spin components impinging on two orthogonal directions. According to this law, we can associate with spin the objective realities given by the following formulas:

$$m_{ix} = m_0 \cos(\lambda_i) \qquad \text{and} \qquad m_{iy} = m_0 \cos(\lambda_i + \pi/2) \qquad \text{for} \quad i = a, b, c \tag{8}$$

where the value of the constant amplitude m_0 will be completely irrelevant in our arguments concerning the magic minus sign in equation (5).

These expressions describe the Malus attenuation (projection) law for objective and local quantities of the spin. The only thing that cannot be controlled during a measurement is the random orientation of these objective quantities. This randomness is described by a set of local hidden angles λ_a, λ_b, and λ_c or spin orientations which have to be averaged with respect to a given distribution $P(\lambda_a; \lambda_b; \lambda_c)$ in order to calculate physical outcomes.

We shall give now a completely *ad hoc* distribution of these angles that will do what quantum mechanics does. We define

$$P(\lambda_a; \lambda_b; \lambda_c) = \frac{1 - \cos(\lambda_a + \lambda_b + \lambda_c)}{(2\pi)^3} \tag{9}$$

Note that this distribution is positive, normalized to unity if integrated over all the hidden angles from 0 to 2π. It also has the right marginals, i.e., any integration over one or two hidden angles produces a uniform distribution of the remaining angles. This property is in full analogy to a reduction of the state (1) to a subspace involving only one or two particles. In such a reduced subspace the state given by equation (1) becomes a completely mixed state.

Using the definitions (6) and the formulas (8) and (9), we obtain

$$O_{1\mathrm{LHV}} = O_{2\mathrm{LHV}} = O_{3\mathrm{LHV}} = -O_{\mathrm{LHV}} \tag{10}$$

i.e., a result identical to the quantum prediction (4). The absolute value of these spin correlations can be made identical to the quantum mechanical result if we set $m_0 = 2$ in the spin objective realities given by equation (8). Of course the local and objective variables in this case do not possess the algebraic properties of Pauli matrices leading to equation (5), but this does not matter. Only quantities averaged over the hidden angle given by equation (6) are observed and can be compared with an actual experiment. The most important conclusion from this model is the fact that the combination of the distribution function (9) with local realities (8) for the three spin-1/2 particles reproduces the magic minus sign in equation (10).

Of course the LHV model presented in this paper cannot be entirely right, because after all we do believe that quantum mechanics is a good theory. This can be seen if the transmission probability through a linear polarizer is investigated. In quantum mechanics the transmission of the particle through a polarizer (along x) is described by a projection operator $\frac{1}{2}(1 + \sigma_{ax})$. In the framework of LHV theory the action of this polarizer can be described, for example, by the LR transmission function $\frac{1}{2}(1 + tm_0 \cos \lambda_a)$, where in analogy to classical optics the parameter t can be interpreted as the visibility of the interference fringes. An everywhere positive LR transmission function with $m_0 = 2$ will lead to a 50% visibility of the LHV correlations.

The quantum mechanical 100% visibility can be reproduced only with a negative transmission function, i.e., in clear violation of LR. But in order to demolish this statistical model and the LR, we have to investigate a series of different experimental outcomes based on Bell's inequality (Wódkiewicz, 1991).

Using different quantum quasidistributions for correlated spin-1/2 particles (Scully and Wódkiewicz, 1990), it is possible to give a unified approach to the problem of locality, reality, and positivity of various physical quantities discussed in this paper. Whatever is the mathematical description of these correlations, the magic minus sign in equation (4) can be achieved in the framework of a statistical LHV model discussed in this paper.

This example shows that the three-particle EPR correlations as derived by Mermin (1990*a*,*b*) can rule out some models of LR, but are not powerful enough to refute the particular model presented in this paper. Thus the very strong statement made by Mermin (1990*a*) that the three-particle *gedanken* experiment is an always vs. never refutation of the EPR argument is premature.

ACKNOWLEDGMENTS

The author would like to thank Prof. M. O. Scully for his invitation and for the hospitality extended to him at the University of New Mexico, where part of this work was done. This work was partially supported by the Office of Naval Research and the KBN program Nr204279101.

REFERENCES

Greenberger, D. M., Horn, M. A., and Zeilinger, A. (1989). In *Bell's Theorem, Quantum Theory and Conceptions of the Universe*, M. Kafatos, ed., Kluwer Academic, Dordrecht, p. 69.

Mermin, N. D. (1990*a*). *Physics Today*, **43**, 9.

Mermin, N. D. (1990*b*). *American Journal of Physics*, **58**, 731.

Mermin, N. D. (1990*c*). *Physical Review Letters*, **65**, 1838.

Scully, M. O., and Wódkiewicz, K. (1990). In *Coherence and Quantum Optics VI*, J. H. Eberly *et al.*, ed., Plenum Press, New York, p. 1047.

Wódkiewicz, K. (1991). To be published.